Perfectly Matched Layer Absorbing Boundary Conditions for the Method of Lines Modeling Scheme

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***Abstract*—The perfectly matched layer device for the absorption of electromagnetic radiation is incorporated with the method of lines numerical scheme. Examples are given that show the superior performance in comparison with numerical absorbing boundary conditions.**

***Index Terms*—Absorbing boundary condition, method of lines, perfectly matched layer.**

I. Introduction

The lossy perfectly matched layer (PML) proposed for the absorption of electromagnetic radiation at all incident angles and independently of polarization [1] has found successful applications in numerical modeling schemes. Much has been written on the numerical implementation, modification, and generalization of the PML device in the last few years, in particular, in truncating the computational windows of the finite-difference time-domain (FDTD) and finite-difference frequency-domain (FDFD) schemes [2]–[4]. In contrast, only a numerical absorbing boundary condition based on the factorization of the Helmholtz operator has been used in the method of lines (MoL) modeling approach [5]. More recently, a heuristic complex factor has been introduced into this condition for the purpose of improving its effectiveness [6]. In this work we describe the incorporation of the perfectly matched layer concept into the MoL numerical scheme. The theory we develop is based on the simultaneous solution of two wave equations, one (the Helmholtz) for the structure being modeled and another (modified Helmholtz) for the PML which is assumed to exist at the boundaries of the computational window. Though the wave equation for the PML derives from the non-Maxwellian split field formalism proposed in [1], the interface boundary conditions we impose are the usual continuity of the tangential field components. As an example we calculate the propagation of a Gaussian pulse in a homogeneous medium and compare the results with both the theoretical predictions and also with those obtained when third-order numerical absorbing boundary conditions replace the action of the PML. We show that agreement with theory is more closely exhibited by the use of the PML. The results are seen to be both consistent and independent of computational window size when the PML device is used.

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II. Theory

Consider the propagation of TE-polarized waves along the z-axis with the field components given by

$$-\frac{\partial H_Z}{\partial x} = (-i\omega\varepsilon + \sigma_x)E_{yx} \tag{1}$$

$$\frac{\partial (E_{yx}+E_{yz})}{\partial z} = (-i\omega\mu + \sigma_z^*)H_x \tag{2}$$

$$\frac{\partial H_x}{\partial z} = (-i\omega\varepsilon + \sigma_z)E_{yz} \tag{3}$$

$$-\frac{\partial (E_{yx}+E_{yz})}{\partial x} = (-i\omega\mu + \sigma_x^*)H_z \tag{4}$$

where $\sigma_{x,\,z}$ and $\sigma_{x,\,z}^*$ account for electric and magnetic losses, respectively. Decoupling the above equations and writing $\varsigma = E_{yx} + E_{yz}$ we obtain the modified Helmholtz equation in the form

$$\frac{1}{\Gamma_x^2}\frac{\partial^2 \varsigma}{\partial x^2} + \frac{1}{\Gamma_z^2}\frac{\partial^2 \varsigma}{\partial z^2} + \omega^2\mu\varepsilon\varsigma = 0 \tag{5}$$

where

$$\Gamma_x^2 = \left(1 + i\,\frac{\sigma_x}{\omega\varepsilon}\right)\left(1 + i\,\frac{\sigma_x^*}{\omega\mu}\right) \tag{6}$$

and

$$\Gamma_z^2 = \left(1 + i\,\frac{\sigma_z}{\omega\varepsilon}\right)\left(1 + i\,\frac{\sigma_z^*}{\omega\mu}\right). \tag{7}$$

It is evident from (5) that, in contrast to the homogeneous medium where $\Gamma_x = \Gamma_z = 1$, plane waves in the PML become inhomogeneous with the real distances x and z transformed into complex distances $x\Gamma_x$ and $z\Gamma_z$, respectively. It has been pointed out that both the amplitude and phase of waves undergoing this kind of transformation remain continuous across the interface separating the homogeneous medium and the PML [7], [8].

III. Results and Discussion

Because of space limitation we will not give a description of the MoL and refer the reader to [5] for an implementation of the method with numerical absorbing boundary conditions. In our present application we consider the propagation of a Gaussian pulse in a homogeneous medium along the z-direction. We assume the Helmholtz equation to apply in the homogeneous medium and (5) in the PML. In order that

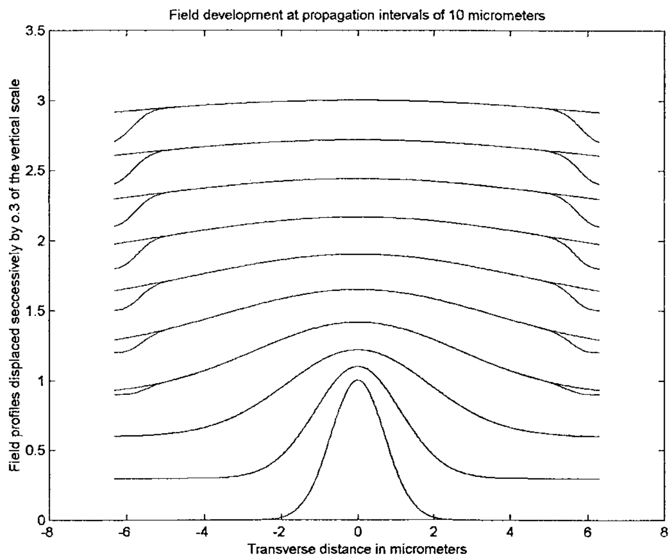

Fig. 1. The development of a Gaussian excitation impressed at $z = 0$. The field profiles shown are at 10-μm intervals. The PML's occupy the end 2 μm on either side of the figure.

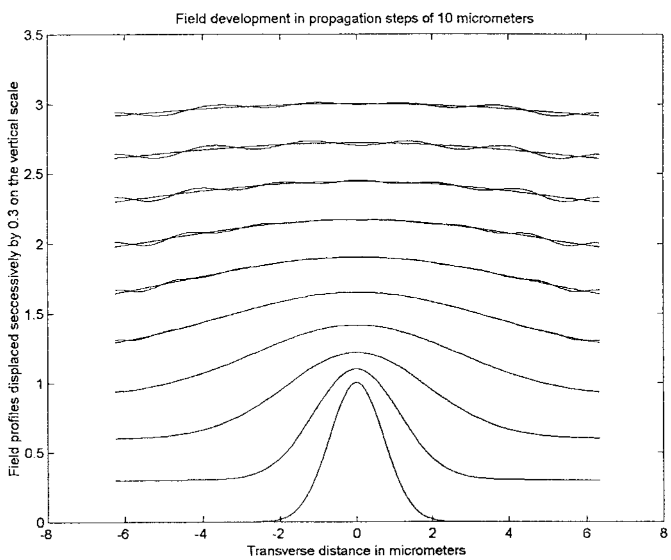

Fig. 2. The same as Fig. 1 except that numerical absorbing boundary conditions are substituted for the PML. This is an example of well-tuned absorbing boundary conditions.

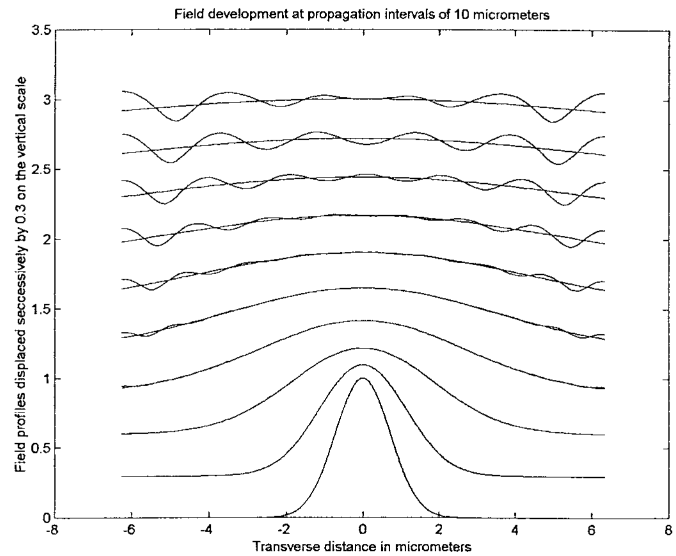

Fig. 3. The same as Fig. 2, but for badly tuned absorbing boundary conditions.

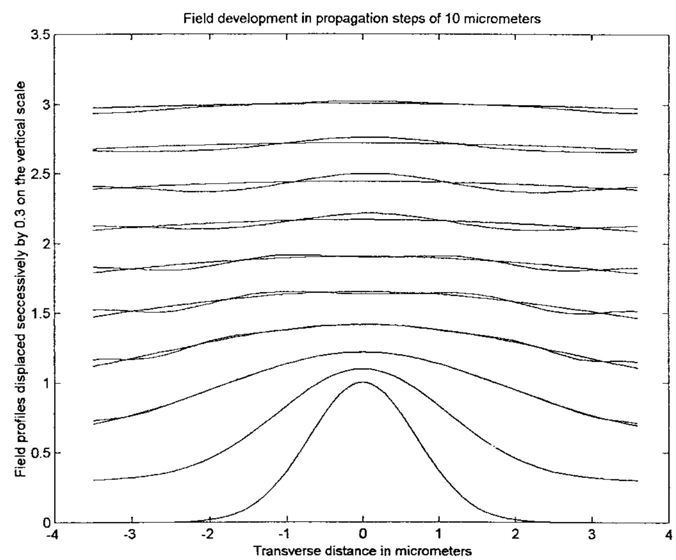

Fig. 4. The same as Fig. 2, but with reduced computation window size.

absorption in the PML takes place in the x-direction only we impose the requirement $\Gamma_z = 1$. The well-known condition

$$\frac{\sigma_x}{\varepsilon} = \frac{\sigma_x^*}{\mu} \tag{8}$$

is upheld so that the impedance of the homogeneous medium is matched to that of the PML and no reflection occurs [1]. Accordingly, (6) is written as

$$\Gamma_x = \left(1 + i\,\frac{\sigma_x}{\omega\varepsilon}\right). \tag{9}$$

The conductivity appearing above has, in fact, been made to increase with distance into the PML according to the relationship $\sigma_x = (m/M)^4$ where m is the line number (increasing into the PML) and M is the total number of lines in the PML. The other parameters used are: wavelength $= 1.3$ μm, the homogeneous medium dielectric constant $= 11.56$ μm, and the width of the PML $= 2$ μm on each boundary of the computational window in the x-direction. The number of lines in PML has been chosen to be 20, which satisfies convergence of results. The excitation applied symmetrically at $z = 0$ has been taken as $\varsigma(0) = e^{-(x/\sigma)^2}$. Fig. 1 shows MoL results together with the theoretical results for a Gaussian excitation with $\sigma = 1$. The PML has been placed at distances of ± 4.35 μm from $x = 0$. The ς profiles have been obtained at z-intervals of 10 μm starting at $z = 0$. They have been shifted successively by 0.3 on the vertical scale for the sake of better visibility. The results of theory and calculation are seen to coincide within the computational window. The only feature that distinguishes the results from each other is the MoL profile tails seen to penetrate into the PML and become attenuated within it. The attenuation, however, does not show

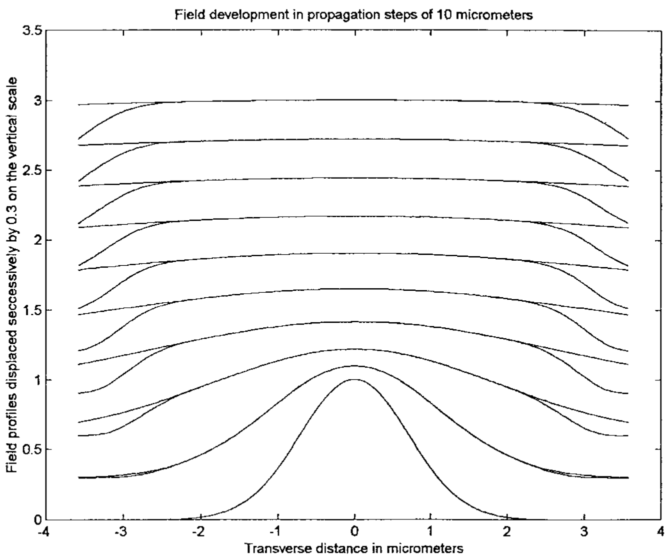

Fig. 5. The same as Fig. 1, but with reduced computation window size.

any selective alteration of the amplitude or phase distributions. The results of Fig. 1 are to be compared with those of Figs. 2 and 3 obtained by using third-order numerical absorbing boundary conditions in place of the PML. All the parameters are otherwise the same as in Fig. 1. The results shown in Figs. 2 and 3 are examples of a well-tuned and badly tuned sets of numerical absorbing boundary conditions, respectively. As is well known, the need to tune these conditions stems from the fact that they do not offer broad-angle transparency in the strict sense, but provide almost perfect absorption at three angles only. The MoL results of Fig. 2 in particular exhibit close agreement with the theoretical profiles and are seen to undulate around them as the Gaussian excitation develops gradually into a plane wave of uniform amplitude. As can be observed this closeness of agreement is, however, both short lived and less than that achieved with the PML. It is, moreover, dependent on the width of the computational window. It is seen to deteriorate as the boundaries are brought closer together. This effect is demonstrated in Fig. 4 in which the computational window has been reduced by 5.5 μm with all other parameters being the same as in Fig. 2. In contrast to this the PML results remain in excellent agreement with theory throughout the propagation distance as shown in Fig. 5.

IV. Conclusions

The incorporation of the PML concept into the MoL modeling scheme has been achieved and its effectiveness demonstrated. The fact that the PML provides reflectionless absorption at all angles and frequencies independently of polarization makes it particularly suited for truncating the computational windows in MoL calculations. The application of the PML, moreover, eliminates the need for tuning the numerical boundary conditions as the wave propagates.

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References

[1] J.-P. Berenger, “A perfectly matched layer for the absorption of electromagnetic waves,” *J. Comput. Phys.,* vol. 114, pp. 185–200, 1994.
[2] D. S. Katz, E. T. Thiele, and A. Taflove, “Validation and extension to three dimensions of the Berenger PML absorbing boundary condition for the FD–TD meshes,” *IEEE Microwave Guided Wave Lett,* vol. 4, pp. 268–270, 1994.
[3] B. Chen, D. G. Fang, and B. H. Zou, “Modified Berenger PML absorbing boundary conditions for FD–TD meshes,” *IEEE Microwave Guided Wave Lett.,* vol. 5, pp. 399–401, 1995.
[4] U. Pekel and R. Mittra, “Mesh truncation in the finite element frequency-domain method with a perfectly-matched layer (PML) applied in conjunction with analytic and numerical absorbing boundary conditions,” *Microwave Opt. Technol. Lett.,* vol. 9, pp. 176–180, 1995.
[5] A. Dreher and R. Pregla, “Analysis of planar waveguides with the method of lines and absorbing boundary conditions,” *IEEE Microwave Guided Wave Lett.,* vol. 1, pp. 138–140, 1990.
[6] K. Wu and X. Jiang, “The use of absorbing boundary conditions in the method of lines,” *IEEE Microwave Guided Wave Lett.,* vol. 6, pp. 212–214, 1996.
[7] C. M. Rappaport, “Perfectly matched absorbing boundary conditions based on anisotropic lossy mapping of space,” *IEEE Microwave Guided Wave Lett.,* vol. 5, pp. 90–92, 1994.
[8] R. Mittra and U. Pekel, “A new look at the perfectly matched layer (PML) concept for the reflectionless absorption of electromagnetic waves,” *IEEE Microwave Guided Wave Lett.,* vol. 5, pp. 84–86, 1995.